Simulation of Nonlinear Vibrations of a Rotor System with Sealing Fluid Forces

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Abstract: The bifurcation and chaotic vibrations of the rotor system are simulated under the condition of the sealing forces excited by labyrinth seals. The rotor system, which is the important part of rotating machinery, will suffer serious self-excited (i.e. fluid induced) vibrations stemming from sealing structures. At first, the dynamic differential equations of the rotor system involving in sealing forces are established, in which the seal forces are determined based on the description of Muszynska (2005) known as a Muszynska sealing model. Then, numerical integration method is used to simulate the nonlinear vibrations of the rotor system under the given structure parameters and initial conditions. The obtained results of the rotor system include motion bifurcation patterns and different vibration behaviors. Based on them, the influences of the rotating speeds on the vibrations of the rotor system are compared to reveal the nonlinearity of the rotor system with sealing fluid forces.

Key-Words: *rotor system; sealing fluid force; bifurcation; self-excited vibration.*

1. Introduction

Virtual prototype of machine in cyberspace is known as a highly developed and critical platform which is powerful to help machine designing, manufacturing and maintenance. In the process of establishing a virtual prototype of a machine, the modeling technologies of CAD/CAE/CAM are important. Among them, the technique of computer-aided-analysis is the most difficult and often limited to mechanic analyses. Recently, the concept of multi-physics simulation belonging to CAE is widely accepted and used steadily in machine designing of higher-tech equipments (Zhou et al., 2010), especially rotating machinery such as aero-engines in which aero-elastic and thermal and structure analyses are integrated.

In practice, the rotor system of rotating machinery with sealing structures (i.e. rotor-sealing system) often suffers serious vibrations (forced and/or self-excited) stemming from the sealing structures. The changing flow and its pressure of the seal cavity easily produce external exciting forces during operating process of the machine, especially considering the sealing aero-elastic property. Different sealing structures may have different influence degree. The exciting effect of sealing structure on rotor vibration can be simplified and described as sealing fluid forces. Mostly, the sealing induced excitation forces are unavoidable and cause serious self-excited vibration of the rotor system.

In order to investigate the vibrations of the rotor system caused by sealing fluid forces, many researches have

Received: 11 February 2012, Accepted: 1 March 2012
Available online 12 March 2012

been achieved for many years (Black et al., 1969, Childs et al., 1978). Childs (1978) and Nelson (1988) studied the sealing fluid forces by both theoretical and experimental analyses. But, these studies are almost limited to the linear stability of the rotor-sealing system. The instability of the rotor system should be analyzed based on nonlinearity of the sealing fluid force. The Muszynska model, which can represent the possible nonlinear mechanism of the rotor-sealing system (Muszynska et al., 1990), is useful to analyze the system instability. The Muszynska model includes the effects of inertia, damping and stiffness of the sealing structure contributing to the rotor system, in which the ratio of average flow velocity is the key parameter to describe the sealing fluid force.

In this paper, based on the Muszynska model of sealing fluid force, numerical simulations are used to inspect the rotor system vibrations caused by sealing fluid forces. At first, the dynamic equations of the rotor system involving Muszynska model are established. Then, numerical integrations are achieved for the transverse vibrations of the rotating shaft. The nonlinear behaviors of the rotor system are compared when with different rotating speeds. The bifurcation patterns, multi-periodic and chaotic vibrations of the rotor system are obtained.

2. Modeling of the rotor system with sealing structure

Shown in Fig. 1, a rotating disc drum with labyrinth sealing structure is mounted on the shaft. The rotor-sealing system is simplified to one which is consisted of a massless, soft-simply-supported shaft and a rigid disc. The disc is regarded as a lumped mass, which is unbalanced with a small eccentricity. The shaft is supported by the two same journal bearings. The two bearings are linearized ideally with stiffness and damping elements.

Only transverse vibrations of the rotor system are considered. The governing equations of the rotor-sealing system are as follows,

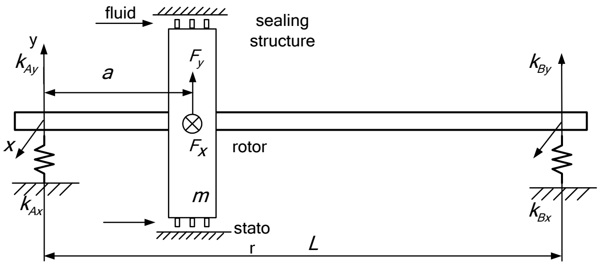


Figure 1 Model of the rotor system

$$\begin{cases} \boldsymbol{M}\ddot{\boldsymbol{u}}_1 + \omega \boldsymbol{J}\dot{\boldsymbol{u}}_2 + \boldsymbol{C}_x\dot{\boldsymbol{u}}_1 + \boldsymbol{K}_x\boldsymbol{u}_1 = \boldsymbol{F}_1 + \boldsymbol{M}_{e1} \\ \boldsymbol{M}\ddot{\boldsymbol{u}}_2 - \omega \boldsymbol{J}\dot{\boldsymbol{u}}_1 + \boldsymbol{C}_y\dot{\boldsymbol{u}}_2 + \boldsymbol{K}_y\boldsymbol{u}_2 = \boldsymbol{F}_2 + \boldsymbol{M}_{e2} \end{cases} \tag{1}$$

where, ω is the rotating speed, $\boldsymbol{M}$, $\boldsymbol{J}$ are the mass and gyroscopic moment matrices, $\boldsymbol{C}_x$, $\boldsymbol{C}_y$ are the damping matrices in *x*- and *y*-directions, $\boldsymbol{K}_x$, $\boldsymbol{K}_y$ are the stiffness matrices, $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\dot{\boldsymbol{u}}_1$, $\dot{\boldsymbol{u}}_2$, $\ddot{\boldsymbol{u}}_1$, $\ddot{\boldsymbol{u}}_2$ are the nodal displacement, velocity and acceleration vectors, respectively, $\boldsymbol{M}_{e1}$, $\boldsymbol{M}_{e2}$ are the unbalance forces. They are given as follows,

$$\boldsymbol{M} = \begin{bmatrix} m & 0 \\ 0 & J_d \end{bmatrix},\ \boldsymbol{J} = \begin{bmatrix} 0 & 0 \\ 0 & J_p \end{bmatrix},\ \boldsymbol{u}_1 = \begin{bmatrix} x \\ \theta_y \end{bmatrix},\ \boldsymbol{u}_2 = \begin{bmatrix} y \\ -\theta_x \end{bmatrix},$$

$$\boldsymbol{K}_x = \begin{bmatrix} k_{11x} & k_{12x} \\ k_{21x} & k_{22x} \end{bmatrix},\quad \boldsymbol{K}_y = \begin{bmatrix} k_{11y} & k_{12y} \\ k_{21y} & k_{22y} \end{bmatrix},$$

$$\boldsymbol{C}_x = \begin{bmatrix} c_{11x} & c_{12x} \\ c_{21x} & c_{22x} \end{bmatrix},\ \boldsymbol{C}_y = \begin{bmatrix} c_{11y} & c_{12y} \\ c_{21y} & c_{22y} \end{bmatrix},\ \boldsymbol{M}_{e1} = \begin{bmatrix} F_{ex} \\ 0 \end{bmatrix},$$

$$\boldsymbol{M}_{e2} = \begin{bmatrix} F_{ey} \\ 0 \end{bmatrix},\ \boldsymbol{F}_1 = \begin{bmatrix} F_x \\ 0 \end{bmatrix},\ \boldsymbol{F}_2 = \begin{bmatrix} F_y \\ 0 \end{bmatrix}.$$

where, x , y are the displacements of the transverse vibrations at the disc center, m is the equivalent lumped mass of the disc, $J_{\mathrm{p}} = \frac{1}{2}mR^2$, $J_{\mathrm{d}} = \frac{1}{2}J_{\mathrm{p}}$. The unbalance forces are $F_{ex} = m_e r\omega^2\cos(\omega t)$, $F_{ey} = m_e r\omega^2\sin(\omega t)$, in which m_e , r are unbalance mass and eccentricity of the disc. F_x , F_y are components of the sealing force shown in next section. The stiffness matrices can be deduced by the corresponding flexibility matrices: $k_{11x} = \frac{a_{22x}}{\Delta}, k_{22x} = \frac{a_{11x}}{\Delta},$

$$k_{21x} = k_{12x} = -\frac{a_{21x}}{\Delta}, \quad \Delta = a_{11x}a_{22x} - a_{12x}a_{21x}.$$

where

$$a_{11x} = \frac{1}{k_{Ax}}(1-\frac{a}{L})^2 + \frac{1}{k_{Bx}}\left(\frac{a}{L}\right)^2 + \frac{4}{3k_c},$$

$$a_{22x} = \frac{1}{L^2}(\frac{1}{k_{Ax}} + \frac{1}{k_{Bx}} + \frac{9}{k_c}),$$

$$a_{21x} = a_{12x} = \frac{1}{L}\left[\frac{a}{k_{Bx}L} - \frac{1}{k_{Ax}}\left(1-\frac{a}{L}\right) + \frac{2}{k_c}\right], \quad k_c = \frac{81EI}{L^3}.$$

3. The Muszynska model of sealing fluid forces

In Muszynska sealing model, the sealing fluid and its force vector are assumed to be rotating at $\tau\omega$, where τ is the average velocity ratio of the fluid in circumferential direction. The components of the sealing fluid force are as follows (Childs, 1978),

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -\begin{bmatrix} K - m_f\tau^2\omega^2 & \tau\omega D \\ -\tau\omega D & K - m_f\tau^2\omega^2 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} - \begin{bmatrix} D & 2\tau m_f\omega \\ -2\tau m_f\omega & D \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} - \begin{bmatrix} m_f & 0 \\ 0 & m_f \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \quad (2)$$

where, D is viscous damping coefficient, and K is radial stiffness of the fluid that flows through the sealing clearance, respectively; m_f is the effective mass of the sealing fluid. The coefficients K, D, τ are the nonlinear functions of x, y as follows

$$K = K_0(1-e^2)^{-n}, D = D_0(1-e^2)^{-n}, \quad \tau = \tau_0(1-e^2)^{-b} \quad (3)$$

where, e is the relative eccentricity of rotors, $e = \sqrt{x^2+y^2}/c$, c is the sealing clearance. $K_0 = \mu_0\mu_3$, $D_0 = \mu_1\mu_3 T$, $m_f = \mu_2\mu_3 T^2$, in which

$$\mu_0 = \frac{2\sigma^2}{1+z+2\sigma}E(1-m_0),$$

$$\mu_1 = \frac{2\sigma^2}{1+z+2\sigma}\left[\frac{E}{\sigma} + \frac{B}{2}(\frac{1}{6}+E)\right],$$

$$\mu_2 = \frac{\sigma(\frac{1}{6}+E)}{1+z+2\sigma},$$

$\mu_3 = \frac{\pi R\Delta P}{\lambda}$. And $T = \frac{l}{v_a}, \sigma = \frac{\lambda l}{c}$,

$$\lambda = n_0 R_a^{m_0}[1+(\frac{R_v}{R_a})^2]^{\frac{1+m_0}{2}}, \quad E = \frac{1+z}{1+z+2\sigma},$$

$$B = 2 - \frac{(\frac{R_v}{R_a})^2 - m_0}{(\frac{R_v}{R_a})^2 + 1}, \quad R_v = \frac{Rc\omega}{\gamma}, \quad R_a = \frac{2v_a c}{\gamma}.$$

The parameters used here also include the sealing pressure drop ΔP, inlet loss coefficient z, dynamic viscosity coefficient γ, friction loss gradient coefficient σ, Reynolds number R_v and R_a (in axial flow), axial velocity of fluid v_a, length and radius of the sealing l, R. Coefficients m_0, n_0 should be determined from experimental measurement. The empirical values of n, b, τ_0 are determined by the sealing structure and experiments too, for example, $n = 0.5 \sim 3, 0 < b < 1$, $\tau_0 < 0.5$ (Zhang, 1990).

4. Simulation examples

In simulations, the structure parameters of the rotor-sealing system shown in Fig. 1 are: d=30mm, L=300mm, d_0 =240mm, b=25mm, ρ = $7.85\times10^3\,\mathrm{kg/m^3}$, m_e =95g, r=2mm, $K_{Ax} = K_{Bx} = 3\times10^5$ N/m, a=100mm, $K_{Ay} = K_{By} = 9\times10^5$ N/m .

The parameters of the sealing structure are: m_f =20kg, c=1.5mm, K=7.57×10^5N/m, R=100mm, L=25mm, ΔP=0.75MPa, γ=1.3×10^{-3}Pa.s, v_a=5 m/s.

The other values of the used parameters in simulations include: m_0= -0.25, n_0=0.079, τ_0=0.5, z=1.5, n=2, b=0.5.

The bifurcation diagram of the transverse vibrations of the rotor-sealing system along the rotating speeds is shown in Fig. 2. It can be seen that, in the low rotating speed range of f<155Hz, the rotor system is in a stable single-periodic motion. Otherwise, if the rotating speed is up to 155Hz and more, the rotor system becomes unstable.

The frequency spectra, Poincare mapping and axis center orbits under different rotating speeds are shown in Fig. 3, Fig. 4, and Fig. 5.

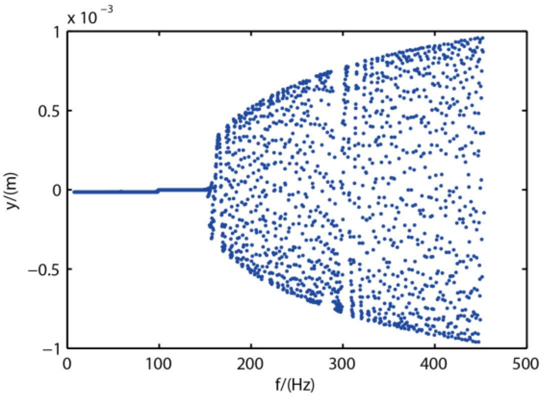
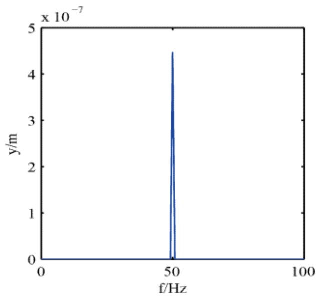
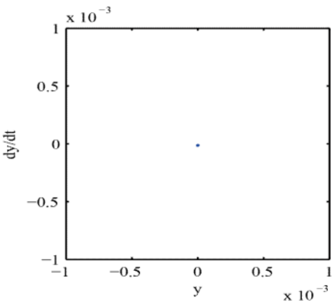


Figure 2 Bifurcation diagram of the rotor-sealing system along the rotating speeds

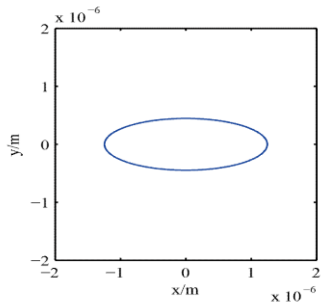
From Fig. 3 to Fig. 5, it can also be seen that, when the rotating speed is below 155Hz, the system is in a single steady state of motion. When the rotating speed is up to 155Hz or more, the Poincare mapping diagram becomes a closed curve, and it implies that there are multiple frequency components of the shaft vibrations. It is also demonstrated sin the obtained frequency spectra and axis center orbits. The simulation results show that there are so many complicated vibration patterns of the system suffering from quasi-periodic motions to chaotic motions when the rotating speeds increase.



(a) Frequency spectrum

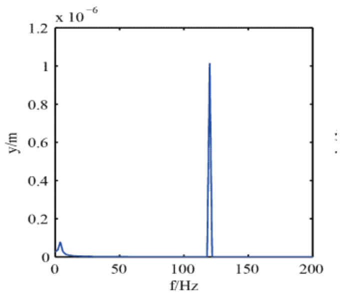


(b) Poincare mapping

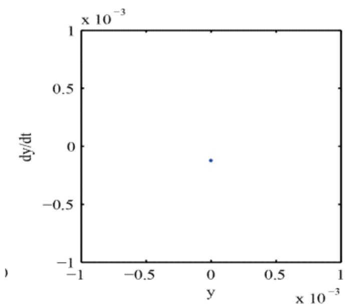


(c) Axis center orbit

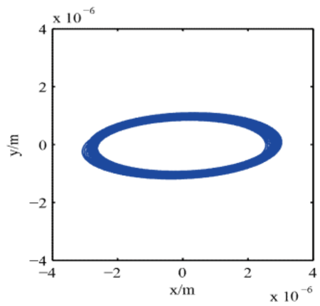
Figure 3 Responses of the system at rotating speed f=50Hz



(a) Frequency spectrum

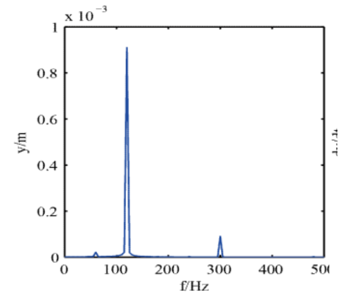


(b) Poincare mapping

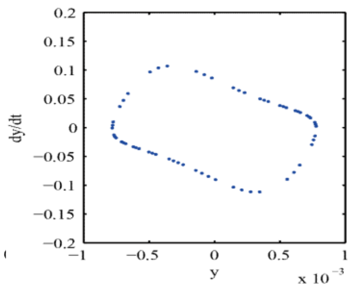


(c) Axis center orbit

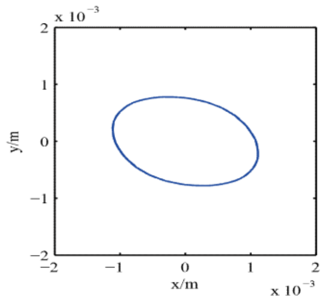
Figure 4 Responses of the system at rotating speed f=120Hz



(a) Frequency spectrum



(b) Poincare mapping



(c) Axis center orbit

Figure 5 Response of the system at rotating speed f=300Hz

5. Conclusions

The vibrations of the rotor system with sealing fluid forces are simulated based on the numerical integration of the governing differential equations of the rotor-sealing system involving Muszynska sealing model. The obtained bifurcation and Poincare mapping diagrams show that the vibrations of the rotor-sealing system is nonlinear including periodic, quasi-periodic and even chaotic motions, when the rotating speeds change.

Acknowledgement

The work was supported by Natural Science Foundation of China (Grant No. 51175070) and the Fundamental Research Funds for the Central Universities (Grant No. N090603007).

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